Pacific Graphics 2015
N. J. Mitra, J. Stam, and K. Xu
(Guest Editors)

Volume 34 (2015), Number 7

Virtual Spherical Gaussian Lights for Real-time Glossy Indirect Illumination (Supplemental Material)

Yusuke Tokuyoshi†

Square Enix Co., Ltd., Japan

Appendix A: Spherical Gaussian Approximation for Reflection Lobes

Diffuse lobes. For the Lambert BRDF ρ_d, the diffuse lobe can be approximated with a spherical Gaussian taking energy conservation into account as follows:

$$\boxed{\rho_d(\mathbf{x},\boldsymbol{\omega}',\boldsymbol{\omega})\langle\boldsymbol{\omega},\mathbf{n}\rangle = R_d\frac{\langle\boldsymbol{\omega},\mathbf{n}\rangle}{\pi} \approx R_d\frac{G(\boldsymbol{\omega},\mathbf{n},\lambda_d)}{A(\lambda_d)}}, \quad \text{(A.1)}$$

where $\langle\boldsymbol{\omega},\mathbf{n}\rangle = \max(\boldsymbol{\omega}\cdot\mathbf{n},0)$, R_d is the diffuse reflectance, and $\lambda_d \approx 2$ which is obtained by using the least square method.

Specular lobes. For the microfacet BRDF ρ_s, the specular lobe is fitted with a single spherical Gaussian by using Wang's analytical approximation [WRG*09]. The BRDF is separated into two factors: the normal distribution function without a normalization factor $D(\boldsymbol{\omega}_h)$ and the rest of the factors $M(\boldsymbol{\omega})$ as follows:

$$\rho_s(\mathbf{x},\boldsymbol{\omega}',\boldsymbol{\omega})\langle\boldsymbol{\omega},\mathbf{n}\rangle = M(\boldsymbol{\omega})D(\boldsymbol{\omega}_h),$$

where $\boldsymbol{\omega}_h$ is the half-way vector of $\boldsymbol{\omega}'$ and $\boldsymbol{\omega}$. Bell-shaped normal distribution functions (e.g., Phong [Bli77], Beckmann [BS63] and GGX distributions) can be approximated with a spherical Gaussian as

$$D(\boldsymbol{\omega}_h) \approx G(\boldsymbol{\omega}_h,\mathbf{n},\lambda_h).$$

For Beckmann or GGX normal distribution functions, $\lambda_h = \frac{2}{\alpha^2}$ where α is the roughness parameter. Using spherical warping, this can be approximated with a function of $\boldsymbol{\omega}$ as

$$G(\boldsymbol{\omega}_h,\mathbf{n},\lambda_h) \approx G(\boldsymbol{\omega},\boldsymbol{\xi}_s,\lambda_s),$$

where $\boldsymbol{\xi}_s$ is the reflection vector given by $\boldsymbol{\xi}_s = 2(\boldsymbol{\omega}'\cdot\mathbf{n})\mathbf{n} - \boldsymbol{\omega}'$, and $\lambda_s = \frac{\lambda_h}{4|\boldsymbol{\omega}'\cdot\mathbf{n}|}$. Hence, the specular lobe is approximated with the following equation:

$$\rho_s(\mathbf{x},\boldsymbol{\omega}',\boldsymbol{\omega})\langle\boldsymbol{\omega},\mathbf{n}\rangle \approx M(\boldsymbol{\omega})G(\boldsymbol{\omega},\boldsymbol{\xi}_s,\lambda_s).$$

Since microfacet BRDFs almost preserve energy for highly glossy surfaces, this paper moreover approximates the specular lobe using a normalized spherical Gaussian as follows:

$$\boxed{\rho_s(\mathbf{x},\boldsymbol{\omega}',\boldsymbol{\omega})\langle\boldsymbol{\omega},\mathbf{n}\rangle \approx R_s\frac{G(\boldsymbol{\omega},\boldsymbol{\xi}_s,\lambda_s)}{A(\lambda_s)}},$$

where R_s is the specular reflectance. anisotroic spherical Gaussians are also usable in the same manner [XSD*13].

Appendix B: Shading via Product Integrals of Spherical Gaussians

Diffuse reflection. Using Eq. 5 of the main document and Eq. A.1, the rendering integral of the diffuse component can be calculated using the analytical product integral of two spherical Gaussians. However, Eq. A.1 can produce light leak errors. Unlike the secondary bounce represented by virtual spherical Gaussian lights (VSGLs), light leaks are noticeable at the first bounce which is more visually important. Therefore, the cosine factor is assumed to be a constant and pulled out of the integral [WRG*09] as follows:

$$\begin{aligned} L_d(\mathbf{x}_p,\boldsymbol{\omega}_p) &= \int_{S^2} L_{in}(\mathbf{x}_p,\boldsymbol{\omega})\rho_d(\mathbf{x}_p,\boldsymbol{\omega}_p,\boldsymbol{\omega})\langle\boldsymbol{\omega},\mathbf{n}_p\rangle \mathrm{d}\boldsymbol{\omega} \\ &\approx \frac{c_{in}R_d}{\pi}A(\lambda_{in})\langle\boldsymbol{\xi}_{in},\mathbf{n}_p\rangle. \end{aligned}$$

In addition, when λ_{in} is not small, $A(\lambda_{in}) \approx \frac{2\pi}{\lambda_{in}}$ can be assumed [IDN12]. Therefore, diffuse reflection is inexpensively calculated using the following equation:

$$\boxed{L_d(\mathbf{x}_p,\boldsymbol{\omega}_p) \approx \frac{2c_{in}R_d}{\lambda_{in}}\langle\boldsymbol{\xi}_{in},\mathbf{n}_p\rangle}.$$

Specular reflection. While spherical Gaussians are used for VSGLs, this paper employs an anisotropic spherical Gaussian to approximate a specular lobe at a shading point. This is because a specular lobe can be anisotropic even if it is an isotropic BRDF model, especially for shallow grazing angles. For simplicity, anisotropic spherical Gaussians are used only for the first bounce which is more visually important than the second bounce. In addition, the product integral of a spherical Gaussian and anisotropic spherical Gaussian [XSD*13] has a reasonable computation cost. An anisotropic spherical Gaussian is defined as

$$\acute{G}(\boldsymbol{\omega}, \boldsymbol{\xi}_x, \boldsymbol{\xi}_y, \boldsymbol{\xi}_z, \eta_x, \eta_y) = \langle \boldsymbol{\omega}, \boldsymbol{\xi}_z \rangle \mathrm{e}^{-\eta_x(\boldsymbol{\omega}\cdot\boldsymbol{\xi}_x)^2 - \eta_y(\boldsymbol{\omega}\cdot\boldsymbol{\xi}_y)^2},$$

where $\boldsymbol{\xi}_x, \boldsymbol{\xi}_y, \boldsymbol{\xi}_z$ are orthonormal vectors, and η_x, η_y are the bandwidth parameters. Since a specular lobe is approximated with an anisotropic spherical Gaussian as $\rho_s(\mathbf{x}_p, \boldsymbol{\omega}_p, \boldsymbol{\omega})\langle \boldsymbol{\omega}, \mathbf{n}_p \rangle \approx M(\boldsymbol{\omega})\acute{G}(\boldsymbol{\omega}, \boldsymbol{\xi}_x, \boldsymbol{\xi}_y, \boldsymbol{\xi}_z, \eta_x, \eta_y)$, the rendering integral is calculated as

$$\begin{aligned} L_s(\mathbf{x}_p, \boldsymbol{\omega}_p) &= \int_{S^2} L_{in}(\mathbf{x}_p, \boldsymbol{\omega})\rho_s(\mathbf{x}_p, \boldsymbol{\omega}_p, \boldsymbol{\omega})\langle \boldsymbol{\omega}, \mathbf{n}_p \rangle \mathrm{d}\boldsymbol{\omega} \\ &\approx \boxed{\frac{\pi c_{in} M(\boldsymbol{\xi}_{in}) \acute{G}\left(\boldsymbol{\xi}_{in}, \boldsymbol{\xi}_x, \boldsymbol{\xi}_y, \boldsymbol{\xi}_z, \frac{\eta_x \nu}{\eta_x + \nu}, \frac{\eta_y \nu}{\eta_y + \nu}\right)}{\sqrt{(\eta_x + \nu)(\eta_y + \nu)}}}, \end{aligned}$$

where $\nu = \frac{\lambda_{in}}{2}$.

References

[Bli77] BLINN J. F.: Models of light reflection for computer synthesized pictures. *SIGGRAPH Comput. Graph. 11*, 2 (1977), 192–198. 1

[BS63] BECKMANN P., SPIZZICHINO A.: *Scattering of Electromagnetic Waves from Rough Surfaces*. MacMillan, 1963. 1

[IDN12] IWASAKI K., DOBASHI Y., NISHITA T.: Interactive biscale editing of highly glossy materials. *ACM Trans. Graph. 31*, 6 (2012), 144:1–144:7. 1

[WRG*09] WANG J., REN P., GONG M., SNYDER J., GUO B.: All-frequency rendering of dynamic, spatially-varying reflectance. *ACM Trans. Graph. 28*, 5 (2009), 133:1–133:10. 1

[XSD*13] XU K., SUN W.-L., DONG Z., ZHAO D.-Y., WU R.-D., HU S.-M.: Anisotropic spherical Gaussians. *ACM Trans. Graph. 32*, 6 (2013), 209:1–209:11. 1, 2